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CERTAIN NONLINEAR LAWS OF MANEUVERING A GLIDING WINGED SPACECRAFT WHILE WITHDRAWING IT FROM A CIRCULAR ORBIT TO A LANDING STRIP

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CERTAIN NONLINEAR LAWS OF MANEUVERING A GLIDING WINGED SPACECRAFT WHILE WITHDRAWING IT FROM A CIRCULAR ORBIT TO A LANDING STRIP

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V.S.Vedrov, G.P.Vladichin, A.A.Kondratov, G.L.Romanov, V.M.Shalaginov

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The problem of guiding a winged spacecraft from a circular orbit to a ground landing strip through a "corridor" in the H, V coordinates is discussed, and nonlinear laws of center-of-mass control are given to solve the problem. Equations are derived for control of lateral and longitudinal motion. Optimum curves are plotted for descent along the corridor, and an automatic control system for withdrawal of the craft to a landing site is given, with a block diagram. The derived control law, with continuous correction of the reversal point, ensures an exact trajectory of motion and gliding at maximum banking angle.

Author

INTRODUCTION

The principal difficulties involved in guiding a winged spacecraft to a landing strip following its descent from a circular orbit consist in the following:

1. The velocity of the vehicle must be reduced, by means of energy dissipation, from escape velocity to a speed close to landing speed.

2. The initial scatter of parameters (distance, altitude) which, on reentry into the denser layers of the atmosphere (70 - 80 km), may reach several hundred

* Numbers in the margin indicate pagination in the original foreign text.

kilometers must be reduced to extremely small errors of the order of several hundred meters.

3. The maneuvering possibilities of the vehicle are extremely restricted: in the upper layers of the atmosphere, by the insufficiency of aerodynamic forces and in the lower layers, by problems of strength of material and heating. The "corridor" in the coordinates (H,V) along which the vehicle must be guided is very narrow.

The present study is concerned with certain nonlinear laws of center-of-mass control which make it possible to overcome these difficulties. The problems of obtaining the necessary information are not considered here. The availability of on-board or ground-based equipment acquiring and processing the corresponding information and producing control signals transmitted to the autopilot, is assumed a priori.

Part I of this report is concerned with longitudinal motion and Part II, with the control of lateral motion.

I. CONTROL OF LONGITUDINAL MOTION /3

Section 1. Control of Longitudinal Motion and General Considerations Concerning the Trajectory of Descent

Flight distance is examined here as a function of the initial conditions and the regime of descent. The equations of motion of a winged spacecraft in the longitudinal plane are (neglecting the Earth's rotation):

$$m \frac{dv}{dt} = -CxSq - \sigma \sin \theta$$

$$mV \frac{d\theta}{dt} = CySq - \sigma(1 - \bar{V}^2) \cos \theta$$

$$\frac{dH}{dt} = V \sin \theta$$

$$\frac{dL}{dt} = V \cos \theta \tag{1}$$

where

$$\overline{V}^2 = \frac{V^2}{gr} \; ; \; r = Ro + H; \; g = \frac{\mu}{r^2}$$

$$Ro = 6371.2 \text{ km}; \; \mu = 3.986 \times 10^5 \text{ km}^3/\text{sec}^2$$

From the system of equations (1) the following relation may be derived:

$$KdE + (1 - \overline{V}^2)\, dL + \frac{V^2}{g}\, d\theta = 0 \tag{2}$$

where

$K = Cy/Cx$ = aerodynamic quality, ($Cy = C_L$; $Cx = C_D$)

$E = H + V^2/rg$ = specific mechanical energy of the spacecraft (total energy with respect to unit weight).

Whence

$$L = \int_{E_e}^{E_s} \frac{K}{1 - \overline{V}^2}\, dE + \int_{\theta_e}^{\theta_s} \frac{V^2}{g(1 - \overline{V}^2)}\, d\theta \tag{3}$$

where E_s and θ_s are the unit mechanical energy of the vehicle and the trajectory angle respectively, at the start of descent, 4

E_e and θ_e are the values of these parameters at the end of descent.

The first term in expression (3) determines the flight distance due to the expenditure of mechanical energy and the second term, the distance due to the ballistic effect.

The flight distance, with K = const due to the decrease in kinetic energy, may be calculated from the formula ($r \approx R_0$):

$$L_v = \frac{R_0}{2} K \ln \frac{1 - \overline{V}_e^2}{1 - \overline{V}_s^2}$$

To evaluate the ballistic distance, we compare it with the distance due to the decrease in mechanical energy. As a result, we have

$$\frac{L_\varepsilon}{L_\theta} = \frac{gK_{av}}{V_{av}^2} \frac{H_s - H_e + \frac{V_s^2 - V_e^2}{rg}}{\theta_s - \theta_e} > \frac{K_{av}\left(1 - \frac{V_e^2}{V_s^2}\right)}{r(\theta_s - \theta_e)}$$

Calculations according to this formula for a vehicle with a high quality factor of the order of 2 - 3 show that the ballistic distance accounts for a small portion of the total distance.

When analyzing L_ε it is convenient to utilize a plane on which we can plot the line grids of constant energy, the constant value of the velocity head $q = \rho v^2/r$ = const, the constant temperature or pressure at the critical point, etc. The descent of the spacecraft may also be represented by a specific curve in the coordinates H,V. To accomplish withdrawal of the spacecraft to the landing site, the spacecraft must be made to move along a preset curve in the coordinates H,V. This problem can be solved for a broad class of curves H(V). The permissible curves H(V) of descent should not twice intersect the same /5 curve E = const. The descent curve H(V) must have no sharp inflections, since in practice this would require impermissibly high overloads. Lastly, certain regions on the H,V-plane may be forbidden for a specific spacecraft, either owing to extremely high overloads or owing to excessively high temperatures or owing to smallness of the velocity head. In other words, the curves H(V) should lie within a definite permissible descent corridor.

Assume that the descent curve H(V) has been chosen so as to meet the above requirements. At each point on this curve q, E, the Mach number, and all the aerodynamic characteristics of the vehicle are known. Therefore, Cy, α, Cx and K = Cy/Cx can be determined at each point. As a result, it is possible to construct the function $K/(1 - \overline{V}^2)$ with respect to E and, accordingly, then calculate the approximate flight distance corresponding to the descent along the selected curve H(V). Of special interest is the curve Hm(V); flight along this curve occurs with maximum aerodynamic quality. If the initial and final points of the descent lie on the curve Hm(V), the first term of the integral (3), i.e., the energy distance L_E will reach its maximum on integration along the curve Hm(V), since we have $K < K_{max}$ on any other curve. In other words, achieving the maximum L_E requires flying with maximum aerodynamic quality.

If the spacecraft has an inertial navigation system or some other facilities for measuring flight altitude and flight speed, it is possible to perform the descent along any permissible curve H(V). In particular, the curves $H_u(V)$ /6 and $H_l(V)$, corresponding to descent along the upper and lower boundaries of the descent corridor, respectively, are of special interest. If the curve Hm(V) lies within the descent corridor and nowhere intersects the curves $H_u(V)$ and $H_l(V)$, then a flight along the curves $H_u(V)$ and $H_l(V)$ involves a shorter distance than the regime of descent along the curve ...*.

In view of the above, a series of simple but highly effective schemes for automatic withdrawal of spacecraft to a landing site may be proposed.

Section 2. Automatic Control System for Withdrawal of a Spacecraft to the Landing Site

This system may be separated into two circuits (Fig.1): the inner or auto-

* Notation missing from original.

nomous circuit and the outer or withdrawal circuit. The autonomous circuit is designed to stabilize the rapid angular motion of the vehicle about its center of mass and to stabilize the variation in flight altitude in accordance with variation in speed, i.e., to stabilize a given descent curve H(V). The homing circuit should generate the required curve H(V) with the object of withdrawing the spacecraft onto the landing site. Consider first the autonomous circuit. Let us assume that, on board the spacecraft, the flight altitude and speed are known and, in addition, a device is carried which makes it possible to generate a function of the form

$$\sigma = \sigma(H, V)$$

For example, at low flight speeds a parameter of this kind, continually /7 measured during the flight, may be the dynamic pressure, equal to the difference between the pressure at the critical point and the static pressure. As is known, dynamic pressure chiefly depends on H and M, i.e., on H and V.

The function of the autonomous circuit is to stabilize the principal parameters that determine the state of the craft, in accordance with the signals $\sigma(H, V)$ arriving from the outer circuit. The law of this stabilization is not considered here. Note that in a number of cases it is possible to employ linear stabilization; in general, however, it is desirable to have nonlinear laws of stabilization and, in particular, a self-adjusting autopilot in view of the marked variations in the efficiency of the controls and aerodynamic characteristics owing to the extensive variation in the M number and velocity head. Let us now consider the outer circuit.

Assume that we can select a curve $H_b(V)$ located between the curve Hm(V) and $H_u(V)$ or Hm(V) and $H_l(V)$. It is desirable for this curve to satisfy the

condition $\sigma = \sigma_0$ = const, since in this case programming of σ with respect to V is not required. The selection of the curve H_u or H_l is dictated by specific considerations which are not directly related to the problems considered, for example, the problems of heating or material strength. Below, we will assume that the reference curve $H_0(V)$ is taken between $Hm(V)$ and $H_l(V)$. We use the curve $H_0(V)$ and the corresponding program $\sigma_0(V)$ as the theoretical curve and the program of descent. When flying in accordance with this program, the descent follows a known trajectory and, therefore, the flight distance is known. Assume further that, during the flight, on the basis of known H and V, a value /8
of $E = H + V^2 rg$ is computed on board the vehicle. Since, for a rated trajectory, it is possible to construct the function L with respect to E, which will be denoted by L_{sp} (specified), the control signal σ_{sp} may be shaped according to the deviation of the true distance from the distance specified for the existing margin E according to the formula

$$\sigma_{sp} = f(\Delta L) = f[L - L_{sp}(E)]$$

for which the function $f(\Delta L)$ may have the form shown in Fig.2. A block diagram of the homing system is shown in Fig.1. If, on reentry into the atmosphere, the distance of the vehicle from the landing strip is longer than was calculated, the flight may follow the program $\sigma m(V)$.

The energy in this regime will be dissipated more economically than in flight following the rated program $\sigma_0(V)$. Ultimately, at some instant of time, the true distance to the airport will equal the value specified for the existing energy margin of the spacecraft, whereupon the flight will follow the reference program $\sigma_0(V)$ until an altitude is reached at which the process of leveling with respect to the ground and landing is commenced. The speed at that altitude will be close to the rated speed, no matter what the initial deviations of the distance and velocity vector from their calculated values might be and regardless of the effect of external disturbances. In the event of overflight, a similar picture will be observed. At first the flight will follow the program $\sigma_s(V)$ and then the program $\sigma_0(V)$.

Calculations on a digital computer for investigating such an automatic /9
homing system show that the precision of homing of a vehicle with $K_{max} \approx 2 - 3$ reaches several kilometers.

It is readily seen that this automatic homing system is fundamentally close to optimal from the viewpoint of accomplishing withdrawal and landing of the vehicle in the presence of a maximally broad range of initial pre-withdrawal conditions, since it allows utilizing, to a marked degree of completeness, the maximally possible maneuvering qualities of the spacecraft in the presence of restrictions.

Section 3. Withdrawal of Vehicle to Reference Trajectories

Generally the initial altitude differs markedly from that required for gliding along a specified curve $\sigma(H,V)$. Therefore, at the beginning of descent, the process of transfer to the specified curve must be organized.

The elimination of large initial deviations may be accomplished by stabilizing the specified constants of the controlling parameter. The moments of transfer from one value of the parameter to another depend on the combination of the initial conditions (H_0, V_0, θ_0, and L_0), i.e., on the initial-state vector, and they may be calculated on a digital computer with the aid of a special control algorithm.

Clearly, achieving the optimally rapid transition process is particularly

important when a minimum gliding distance is required, i.e., in flight along $\sigma_s(H,V)$. The following may be chosen as the control parameter: the angle of /10
attack, the overload n_y, the angle of deviation of the control surface δ_b, etc.

Let us consider an elementary case where the overload n_y is taken as the control parameter.

The switching instants of the control parameter may be determined on the basis of the maximum principle.

Calculations of the transition processes indicate that it is possible to simplify the system of equations describing a process of this kind:

$$V = V_0 = \text{const}$$

$$\operatorname{Sin} \theta = 0$$

$$\operatorname{Cos} \theta = 1$$

$$g = g_{av} = \text{const}$$

These simplifications reduce this system of equations to a second-order system.

The maximum principle states that, in this case, the optimal trajectory is obtained when the maximum value of the control parameter of one sign is first specified and then, at a certain time instant t_1, that sign is reversed.

Since the overload n_y possible at extremely high altitudes is low, we assume that $n_y = 0$ along the first segment, while $n_y = n_{y\max}$ along the second segment.

Upon integration of the simplified system of equations, for the time interval over which n_y = const, we have

$$\theta = \frac{g_{av}}{V_0}(n_y + \bar{V}_0^2 - 1)t + \theta_0$$

(cont'd)

$$H = g_{av}(n_y + \bar{V}^2 - 1)\frac{t^2}{2} + V_0\theta_0 t + H_0 \qquad (4)$$

After elimination of t: /11

$$H = \frac{V_0^2}{2g_{av}(n_y + \bar{V}_0^2 - 1)}\theta^2 + S_1$$

where

$$S_1 = H_0 - \frac{V_0^2\theta_0^2}{2g_{av}(n_y + \bar{V}_0^2 - 1)}$$

Figure 3 shows certain phase trajectories corresponding to the process of transition to the curve $\sigma(V,H)$ in the case where $n_y = 0$ and $n_{y_2} = 4$.

It should be borne in mind that, beginning with a certain altitude, the overload $n_{y_2} = 4$ may be achieved only when $\alpha > \alpha_{max}$. In Fig.3 curve 1, obtained for the maximum permissible entry angle $/\theta_0/_{max}$, corresponds to this case.

Curve 2 determines the minimum entry angle $/\theta_0/_{min}$ (in this case the duration of the effect of the overload n_{y_2} is zero).

Solving system (4), we obtain the required duration of effect of the overload n_{y_1} and the total time of the process:

$$t_{1_1} = -\frac{V_0}{g_{av}(n_{y_1} + \bar{V}_0^2 - 1)}\left[\theta_0 + \sqrt{\frac{\theta_0^2(n_y + \bar{V}_0^2 - 1) - \theta_e^2(n_{y_1} + \bar{V}_0^2 - 1)}{n_{y_2} - n_{y_1}} - \frac{2g_{av}}{V_0^2}\,\frac{(H_0 - H_e)(n_{y_1} + \bar{V}_0^2 - 1)(n_{y_2} + \bar{V}_0^2 - 1)}{n_{y_2} - n_{y1}}}\right]$$

$$t_{\Sigma_1} = \frac{V_0}{g_{av}}\,\frac{\theta_e - \theta_0}{n_{y_2} + \bar{V}_0^2 - 1} - \frac{n_{y_1} - n_{y_2}}{n_{y_2} + \bar{V}_0^2 - 1}\cdot t_{1_1}$$

In addition to calculating t_{1_1} and t_{Σ_1}, it is necessary to select the curve of $\sigma(V,H)$ the flight along which will assure the specified gliding distance (see Section 2).

The exact values of t_1 and t_Σ will differ from those calculated from eqs.(5) derived from the simplified equations. The principal error will be due to the inconstancy of speed, which is particularly marked in the presence of considerable velocity heads, i.e., along the second segment.

All the indicated variables [t_1, t_Σ and $\sigma(V,H)$] should be calculated by /12 the method of successive approximations. Calculations show that the curve of $\sigma(V,H)$ was found with sufficient accuracy beginning with the second approximation, and the values of t_1 and t_Σ, beginning with the fourth.

Figure 3 shows the trajectory obtained by this computational method (curve 3).

Figure 4 gives one of the trajectories obtained by using this maneuvering method. As can be seen, the process occurs without overshooting.

As is known, in linear control systems it is impossible to obtain processes of this kind. In this case, the process is either extremely extended or involves overshooting.

Using an optimal system makes it possible to greatly expand the permissible range of initial conditions (θ_0 and L_0) compared with linear control systems.

II. OPTIMAL LAW OF CONTROL FOR LATERAL MOTION /13

Section 1. Synthesis of the Principle of Optimality

The synthesis of optimal lateral control while guiding a gliding object to a landing strip reduces to finding a maneuver for transferring the object from an initial point with the phase coordinates S_0, L_0, ψ (Fig.5) to a final position with the coordinates $S = 0$, $\psi = 0$ in minimum time. The principle of optimality, as construed in this sense, makes it possible to withdraw the object from a maximum range of initial conditions determined by the maneuverability of the object. In accordance with the theorem of existence and uniqueness (Bibl.1) for optimal control there exists a function $\gamma = F(L, S, \psi)$ (γ is the banking angle), which depends only on the current phase coordinates, such that it determines all the optimal trajectories. The reason why optimal control must be found in this form is that, in the course of its travel, the object will experience the effect of various unknown disturbances. The law of control in the proposed form serves for a continual calculation of the new optimal trajectory on taking into account the new initial conditions, thereby offsetting the effect of the disturbances.

The system of differential equations describing the lateral motion of a flying craft will be

$$\begin{aligned} \frac{ds}{dt} &= -V \sin \psi \\ \frac{dL}{dt} &= -V \cos \psi \cos \gamma \\ \frac{de}{dt} &= -V \\ \frac{d\psi}{dt} &= -\frac{g}{V} \sin \gamma \end{aligned} \tag{6}$$

The system (1) was derived on assuming a small inclination angle of the trajectory θ and a small derivative $\frac{d\theta}{dt}$, so that $\sin \theta \approx \theta$, $\cos \theta \approx 1$, and $\frac{d\theta}{dt} \approx 0$; we neglect the effect of the Earth's rotation. /14

In our further discussion, we assume that the banking angle γ which determines the existing lateral overload is restricted to a degree at which the influence of lateral on longitudinal motion is negligibly small. Without presenting calculations, we will point out that this may be achieved if $-\gamma_0 \leq \gamma \leq \gamma_0$ ($\gamma_0 = 0.3$); here it may be assumed that $\sin\gamma \approx \gamma$, $\cos\gamma \approx 1$.

In addition, to linearize the system (1), we assume $\sin\psi \approx \psi$, $\cos\psi \approx 1$. Note that the system may be completely integrated even without this last assumption but the obtained results, as calculations show, differ little from those obtained with this assumption.

After all simplifications, we have

$$\frac{dS}{dL} = \psi$$

$$\frac{d\psi}{dL} = \frac{g}{V^2}\gamma$$

For the system of equations (2) we shall seek the optimal control $\gamma = \gamma(L)$ and such an optimal trajectory with initial conditions $S(L_0) = S_0$, $\psi(L_0) = \psi_0$ as would, at some distance (the coordinate L_e is not fixed), satisfy the ultimate conditions $S(L_e) = 0$; $\psi(L_e) = 0$. On the basis of L.S.Pontryagin's maximum principle (Bibl.1), it may be found that the function ensuring optimality of control with respect to rapidity of maneuver, is defined by the /15 equation

$$\gamma(L) = \gamma_0 \operatorname{sign}(L_1 - L) \tag{8}$$

In this equation, L_1 is the distance corresponding to the instant of reversal of the banking angle. Here, optimal control is determined by a piecewise-constant function which assumes the values $\gamma = \pm\gamma_0$ and has two constancy intervals (if extraneous disturbances are absent).

The right-hand part of the second equation of the system (2) includes the square of velocity, which is essentially a variable. Numerous calculations show that it is possible to find a comparatively simple analytic expression for velocity as a function of distance; most functions of this kind are contained within the limits of the expressions defined:

$$V^2 = k' (L + a) \tag{9'}$$

$$V = k_1^1 (L + a_1) \tag{9''}$$

where k', k_1^1, a, a_1 are parameters serving as functions of the initial gliding distance.

We then find the solution of the system (2), determining the trajectory of motion of the gliding object from initial to final state for a parabolic dependence of the velocity on distance, i.e., we solve jointly eqs.(2), (3), and (4) for the following final conditions:

$$\left.\begin{array}{ll} L = L_0 - \psi = \psi_0 ; & S = S_0 \\ L = L_e - \psi = 0; & S = 0 \end{array}\right\} \tag{10}$$

Since the banking angle has two fixed constancy intervals $\gamma = \gamma_0$ and $\gamma = -\gamma_0$, the trajectory will consist of two segments. Considering the conditions for the joining of solutions at the boundary between these two segments and integrating the system, we obtain /16

$$L_1 = e^{\frac{\psi_0}{k}} (L_0 + a) - a \sqrt{e^{2\frac{\psi_0}{k}} (L_0 + a)^2 - \frac{1}{k} e^{\frac{\psi_0}{k}} \left[(L_0 + a)^2 (\psi_0 + k) - S_0 (L_0 + a) \right]} \tag{11}$$

The coordinates S_0, L_0, ψ_0 may at each time instant be taken as the current coordinates. Therefore, the reversal should be performed at the instant when the computed value of the coordinate L_1 is compared with the current value of the coordinate L_0.

Thus, the control signal arriving at the input of the autopilot is determined by the formula:

$$\gamma_{sp} = \gamma_0 \text{ sign } \phi(S_0, L_0, \psi_0) \qquad (11')$$

where the control function ϕ is

$$\phi(S_0, L_0, \psi_0) = L_0 - L_1 = L_0 + a - e^{\frac{\psi_0}{k}}(L_0 + a) + \sqrt{e^{2\frac{\psi_0}{k}}(L_0+a)^2 - \frac{1}{k}e^{\frac{\psi_0}{k}}\left[(L_0+a)^2(\psi_0+k) - S_0(L+a)\right]} \qquad (12)$$

The instant of reversal corresponds to the equality:

$$\phi(S_0, L_0, \psi_0) = 0$$

The realization of the control law, solving the problem of optimal landing as formulated above may be performed with a circuit including the following links:

a ground-based or on-board device providing the required primary information (L_{meas}, S_{meas});

a computer which, along with generating the control signal, should process the primary information (perform smoothing, differentiation, noise filtration, etc.); /17

an autopilot stabilizing the vehicle with respect to its center of mass and maneuvering it in accordance with instruction signals.

Section 2. Analysis of Maneuverability in the Presence of Perturbations

When shaping the control signal it is necessary to ensure satisfactory accuracy of maneuvering during the action of various disturbances on the system.

The most likely perturbations during lateral maneuvers of the vehicle are: sidewinds, errors in maintaining the specified banking angle, and deviations of the actual pattern of speed variation from that assumed in determining the control function.

The system of equations describing the motion of the vehicle in the field of a constant sidewind directed along the axis may be presented as:

$$\left.\begin{aligned} \frac{dS^*}{dt} &= \psi^* \\ \frac{d\psi^*}{dt} &= \frac{g}{V^2}\left(\gamma + \frac{W}{g}\frac{dV}{dL}\right) \end{aligned}\right| \qquad (13)$$

where

W is the wind speed;

S^*, ψ^* are the parameters of motion in the wind field.

Equation (13) implies that the wind effect is equivalent to the following change in the originally specified banking angle of the vehicle:

$$\Delta\gamma = \frac{W}{g} \cdot \frac{dV}{dL}$$

The trajectories of motion in the coordinates S, L and on the phase plane, that make it possible to determine the maneuverability of the vehicle in the field of a constant sidewind, are shown in Fig.6. It follows from these data that the obtained control law, thanks to the continuous correction of the reversal point, ensures an exact trajectory. The same results are also obtained in the presence of errors in determining the speed and maintaining the banking-angle control specified by this law. Here, in all cases, in the process of control, movement with a maximum banking angle is ensured and, hence, the /18 maneuvering possibilities of the vehicle are completely exploited.

The other coordinates of the reversal point $S_1 = S_1(L_0, S_0, \psi_0)$, $\psi_1 = \psi_1(L_0, S_0, \psi_0)$ may be found in the same way as $L_1 = L_1(L_0, S_0, \psi_0)$. The choice of one parameter or another as the determining parameter should be based on the nature of the initial information and the simplicity of the computer algorithm. Of major interest is the analytic expression establishing the relationship between phase coordinates at points of banking-angle reversal. In case of a linear dependence of speed on distance $V = k_1^1(L + a)$ the relationship between S_1 and ψ_1 is defined by the following expression

$$S_1 = K_1 \ln \frac{\frac{\psi_1 - \psi_0}{k_1}(L_0 + a_1) + 1}{\frac{2\psi_1 - \psi_0}{k_1}(L_0 + a_1) + 1} - \frac{\psi_1 (L_0 + a_1)}{\frac{\psi - \psi_0}{k_1}(L_0 + a_1) + 1} \tag{14}$$

On the basis of eq.(14) the control function will be /19

$$\Omega(L, \psi, S) = S - S_1(L_0, \psi_0, \psi_1) \tag{15}$$

In certain practical cases, the effect of the initial angle ψ_0 may be ignored and the exact relations $S_1(\psi, L_0)$ may be approximated by direct ones:

$$\phi(S, L, \psi) = S - C(L_0)\psi_0$$

Calculations showed that in this case a high accuracy of maneuver is ensured both under normal conditions and in the presence of various perturbations.

In the control function $S - C(L_0)\psi$ the variable ψ may be replaced by the derivative $\dot{S}$, since $\dot{S} \approx V^{(1)}$.

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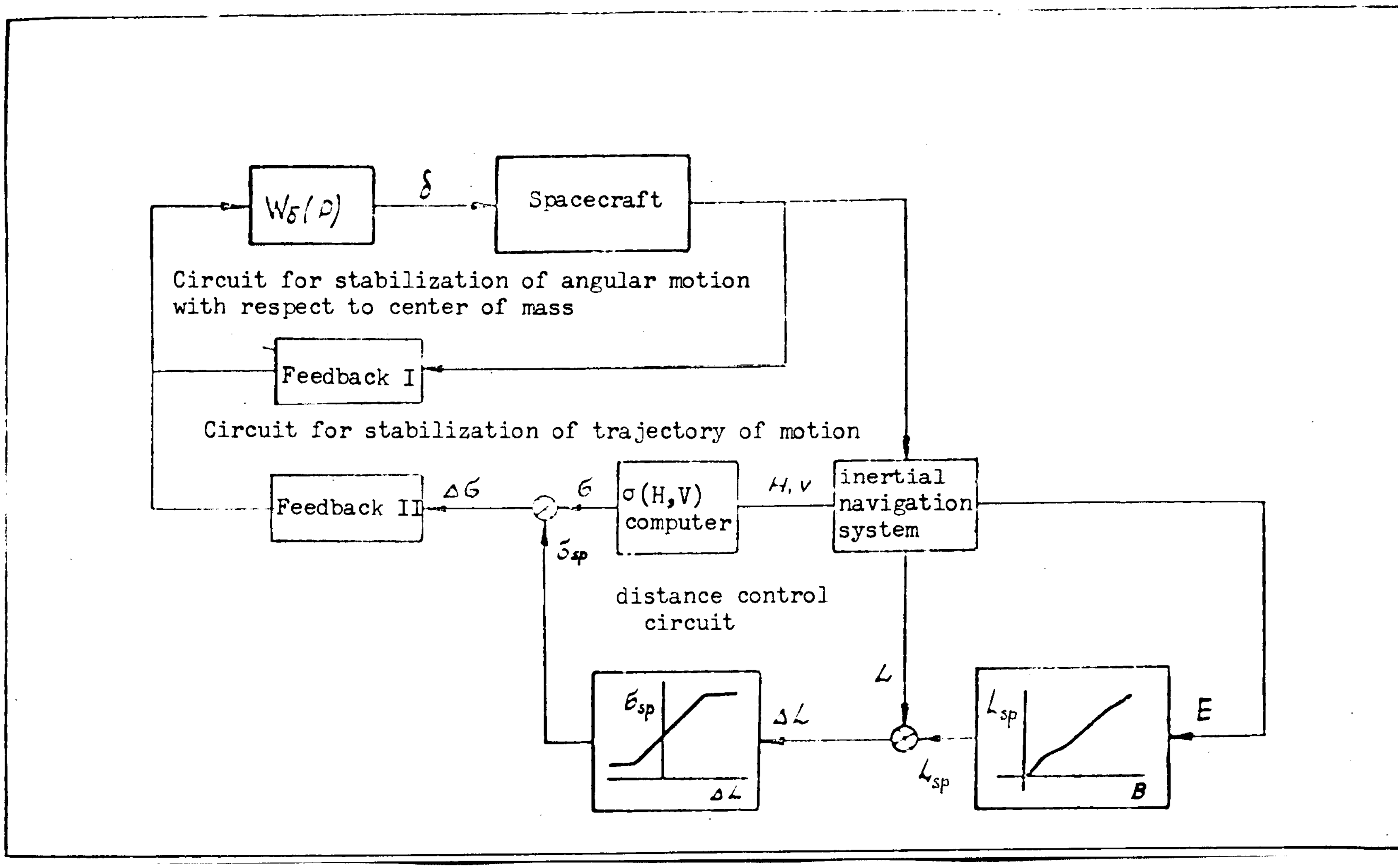

Fig.1 Block Diagram of Control System for Longitudinal Plane

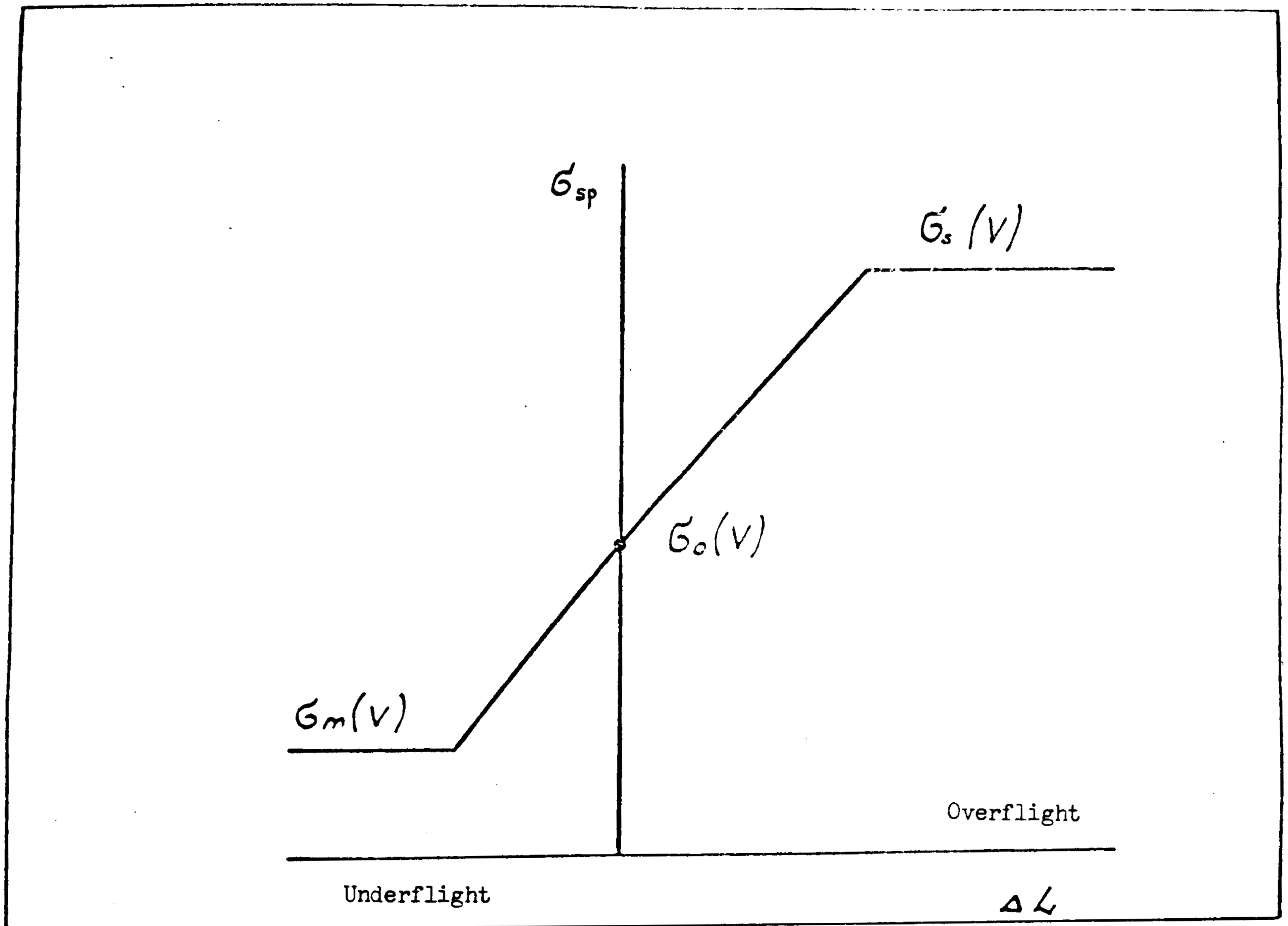

Fig.2 Control Signal as a Function of the Magnitude of Mismatch $\Delta L = L - L_{sp}(E)$

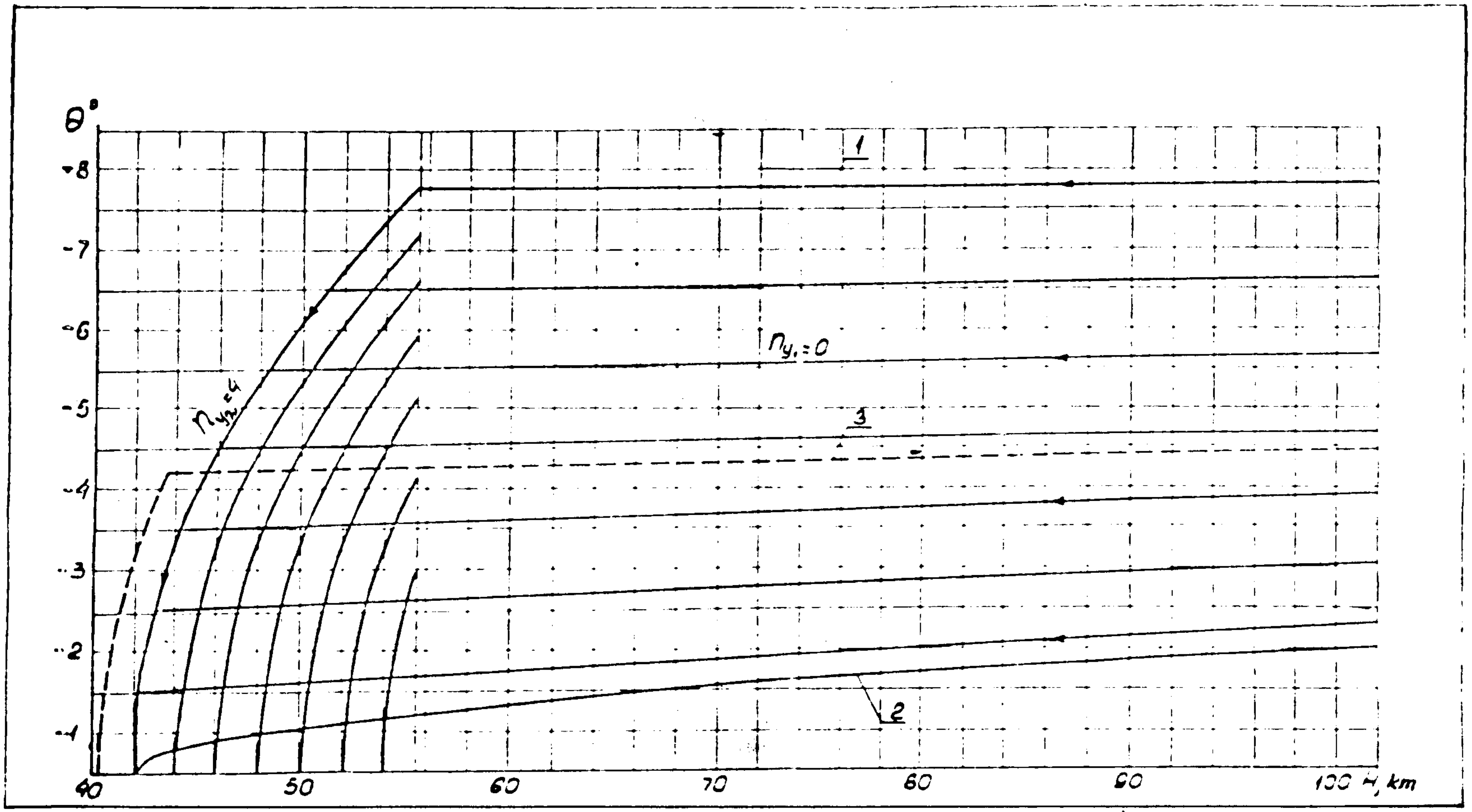

Fig.3 Phase Trajectories of Transition Process to Curve σ(V,H)

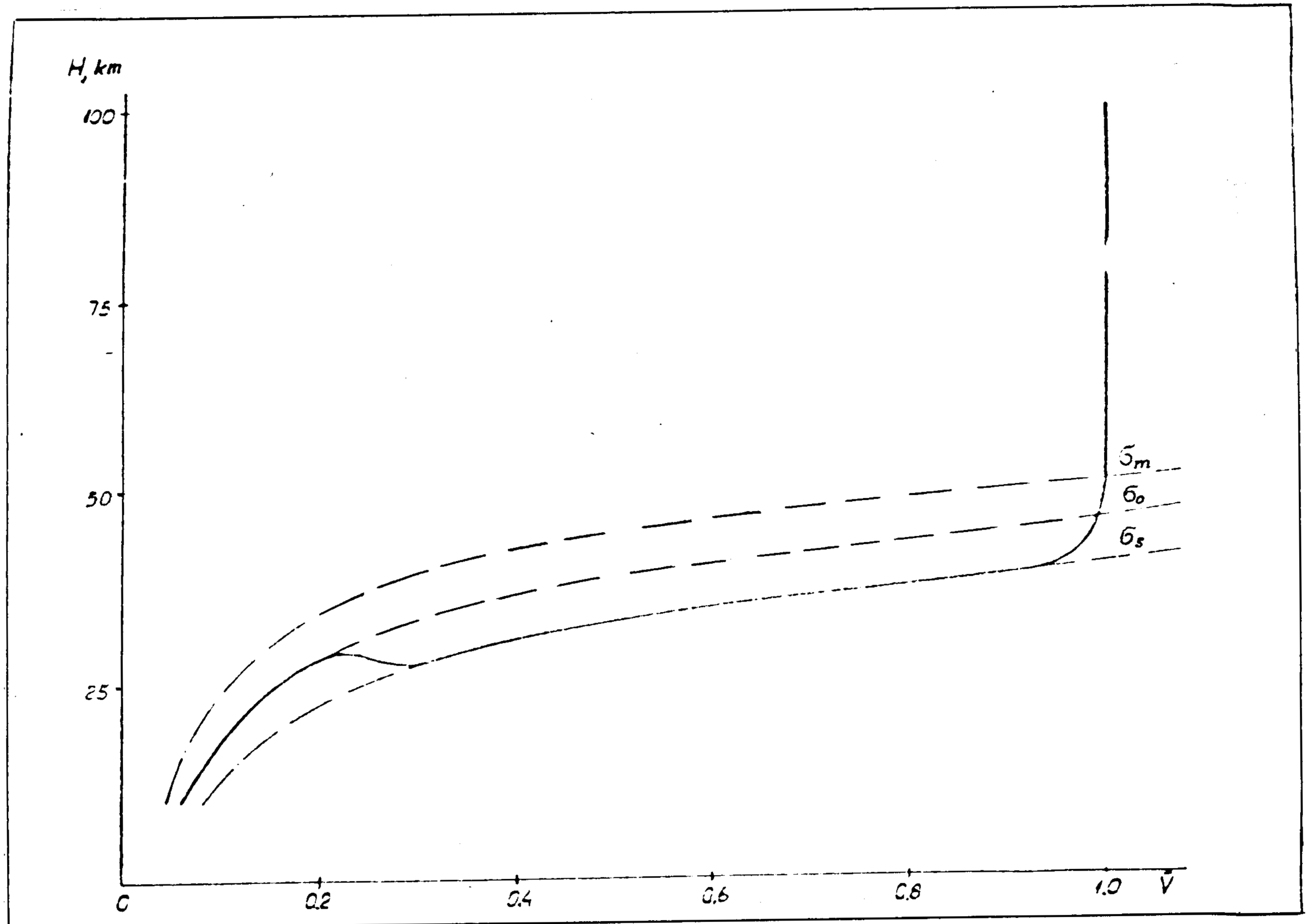

Fig.4 Flight trajectory of the Vehicle in the Coordinates H,V

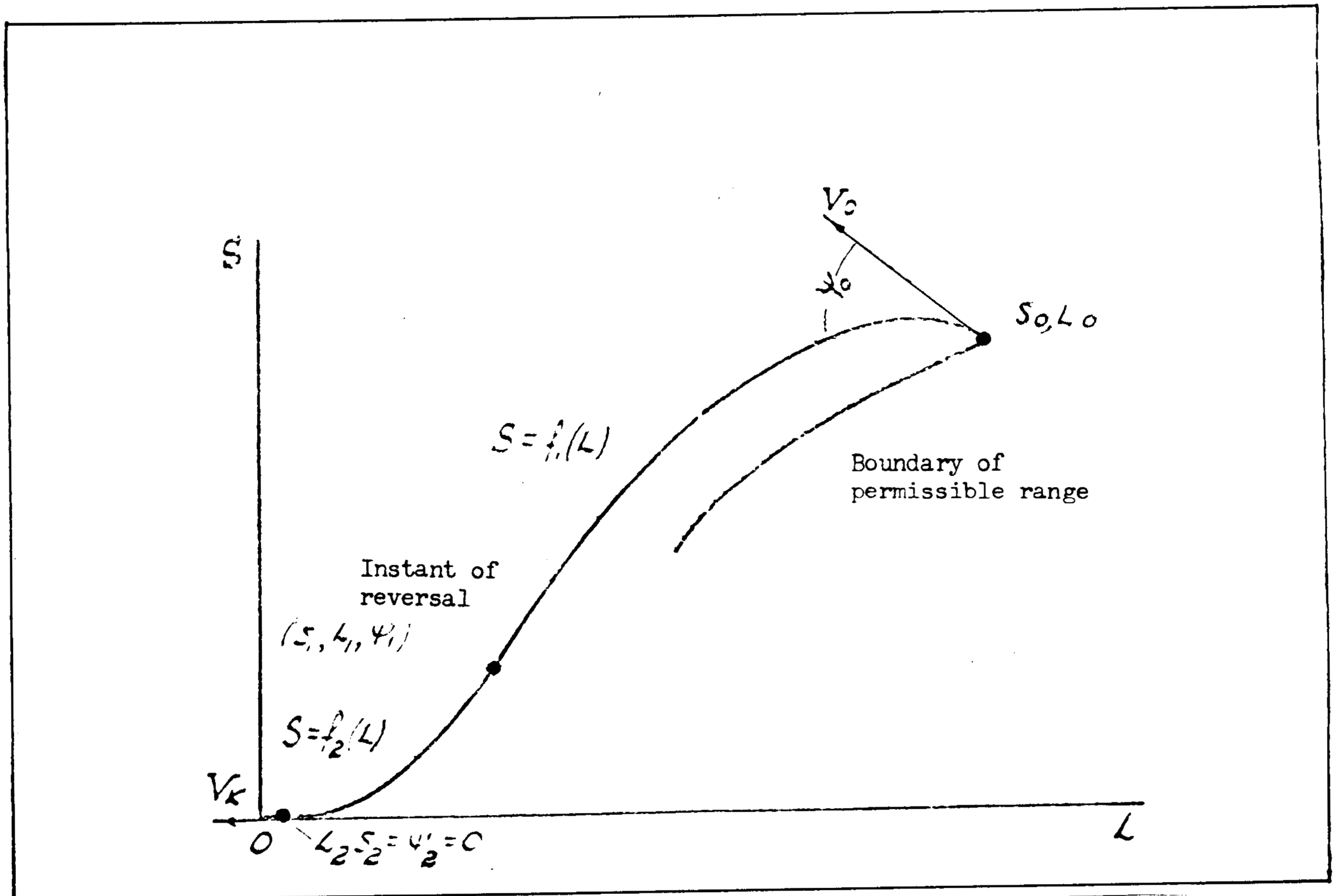

Fig.5 Phase Trajectory in the Horizontal Plane

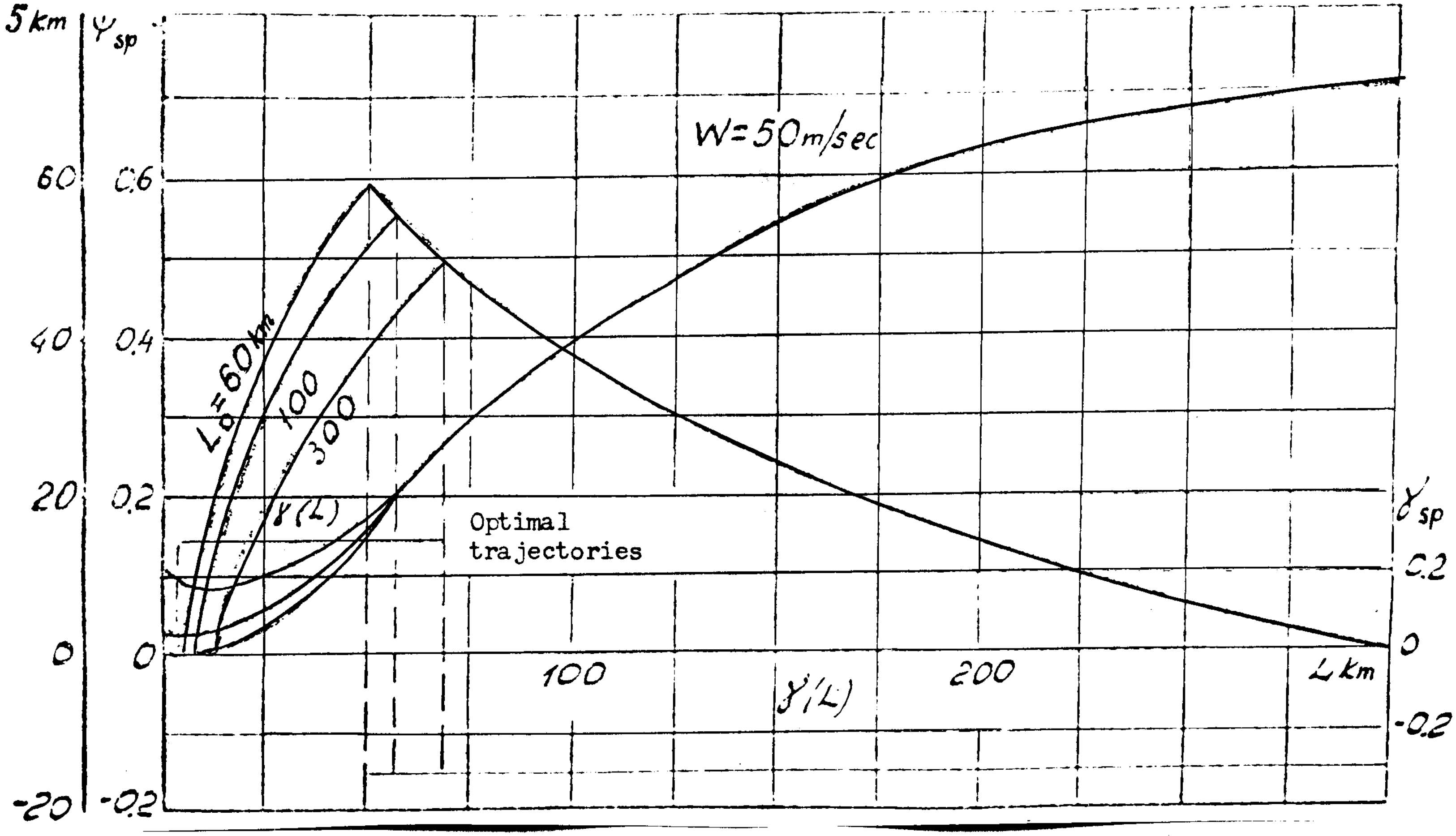

Fig.6 Trajectories of Motion in the Presence of a Constant Wind of a Speed W = 50 m/sec